Economics 212

Section 002

Midterm Exam

October 21, 2014

$30R = 30 \times 126 - 30R$

$60R = 30 \times 126$

$R = 63$

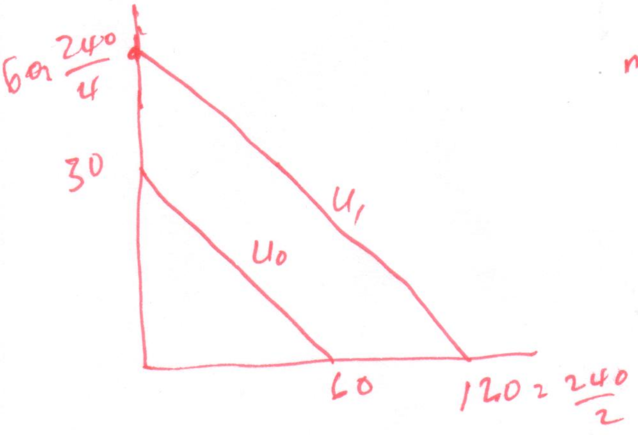
11
14.4
9.6
40

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=2X+5Y, where X and Y are two goods. Draw and appropriately label two indifference curves for this consumer. Assume the price of X is $2, the price of Y is $4 and the consumer has an income of $240. Derive the optimal consumption bundle for the consumer.

$BC = P_x X + P_y Y = 240$



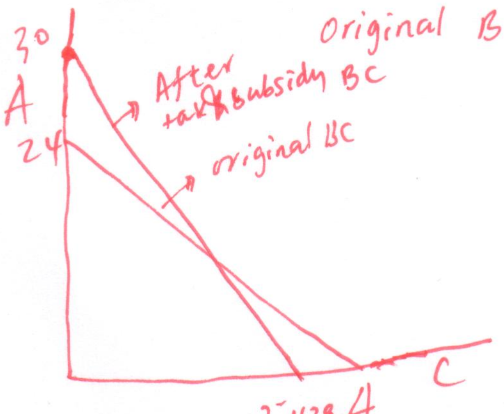
$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{2}{5} = 0.4$ & $\frac{P_x}{P_y} = \frac{2}{4} = 0.5$

$\frac{MU_x}{MU_y} < \frac{P_x}{P_y} \Rightarrow$ So it is optimal to spend all the money on y

$x^* = 0$

$y^* = \frac{240}{4} = 60$

2. [5marks] Marie consumes apples, A, and cheese, C. The price of cheese is $30 per unit and the price of apples is $5 per unit. Marie has an income of $120 to spend on the two goods. Draw and appropriately label her budget constraint. Now suppose the government imposes a tax equal to $5 per unit on cheese and offers a subsidy of $1 per unit on apples. Show how the tax and subsidy scheme affects Marie's consumption opportunities.



Original BC: $30C + 5A = 120$, $C_{max} = 4$, $A_{max} = 24$

After tax and subsidy,

$P_C = 35$ $P_A = 4$ $C_{max} = 3.429$ $A_{max} = 30$

$35C + 4A = 120$

The slope becomes steeper if C is on x axis (and the opposite if C is on y axis)

3. [5marks]A risk averse consumer has a utility of income function given by $U(I)=I^{1/2}$. The consumer has $81 and is asked to reject or accept the following bet: there is a probability of .7 that she will win and finish with an income of $100 and there is a probability of .3 that she will lose and finish with an income of $64. Will the consumer accept or reject the bet? If she accepts the bet, calculate the risk premium associated with the bet.

U_1 Expected utility $0.7 \times (100)^{1/2} + 0.3(64)^{1/2} = 0.7(10) + 0.3(8)$
$= 7 + 2.4 = 9.4$

U_0. utility without the bet is $81^{1/2} = 9$

$U_0 < U_1 \Rightarrow$ The consumer accepts the bet

Expected income with bet is $0.7(100) + 0.3(64) = 89.2$

~~Risk premium = E(I) − E(I)~~
~~= 89.2 − 81.4 = 8.00~~

Risk premium
$.7(10) + .3(8) =$
$U(.7(100) + .3(64) - P)$
$9.4^2 = 89.2 - P$
$P = 89.2 - 9.4^2$
$= 0.84$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Daniel consumes two goods, X and Y, according to the utility function $U(X,Y)=X^{1/2}Y^{1/2}$. The price of X is P_X and the price of Y is P_Y. Daniel has an income I.
   a) [5 marks] Derive Daniel's demand functions for the two goods.

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\frac{1}{2}x^{-1/2}y^{1/2}}{\frac{1}{2}x^{1/2}y^{-1/2}} = \frac{y}{x} \Rightarrow y/x = \frac{P_x}{P_y} \Rightarrow P_y y = P_x x$$

BC $P_x X + P_y y = I$

$2P_x X = I$

$$X^* = \frac{I}{2P_x} \qquad y^* = \frac{I}{2P_y}$$

b) [5 marks] Assume the price of X is \$6, the price of Y is \$3 and that Daniel has an income of \$300. Determine Daniel's optimal bundle.

$X_0^* = \frac{300}{2(6)} = \frac{I}{2P_x}, \quad y^* = \frac{300}{2(3)}$

$X_0^* = \cancel{300}\ 25 \quad y_0^* = 50$

c) [5 marks] Assume that the price of the good X decreases to \$4. Determine the new optimal bundle and the income and substitution effects of the price increase.

$X_1^* = \frac{300}{2(4)} = 37.5 \quad y_1^* = \frac{300}{2(3)} = \underline{50 = X_1^*}$

original utility $= X_0^{*1/2} y_0^{1/2} = 25^{1/2} \cdot 50^{1/2} = 5 \times 5\sqrt{2} = 35.355$

at new price $MRS_{xy} = \frac{y^D}{x^D} = \frac{4}{3} \Rightarrow 3y = 4x$

$X_1^{D^{1/2}} y_1^{D^{1/2}} = 35.355 \Rightarrow x^{D^{1/2}} \cdot \left(\frac{4}{3} x^D\right)^{1/2} = 35.355$

$\sqrt{4/3} \cdot x^D = 35.355$

$\underline{x^D = 40.8244}$

$x^D - X_0^*$

Sub. effect $= 40.8244 - 25 = \underline{15.8244}$

$X_1 - X_1^D$

Inc. effec. $50 - 40.8244 = \underline{9.1756}$

2. Jo has 112 hours per week to divide between leisure, R, and work, L. When she works, Jo earns \$20 per hour. She values both leisure and consumption, C, according to the utility function $U(R,C)=R^{1/2}C$. The price of the consumption good is unity.

   a) [5marks] Derive Jo's optimal bundle. How much does she work?

$$\boxed{C = 20(112-R) \Rightarrow BC}$$

$$MU_R = \tfrac{1}{2}R^{-\frac{1}{2}}C$$
$$MU_C = R^{1/2}$$
$$\Rightarrow \frac{MU_R}{MU_C} = \frac{\frac{1}{2}R^{-1/2}C}{R^{1/2}} = \frac{\frac{1}{2}C}{R} = \frac{20}{1} \quad \frac{w}{1}$$

$$C = 40R$$

$$C = 20(112-R)$$
$$40R = 20(112-R) \Rightarrow R =$$
$$60R = 20 \times 112$$
$$R = 37.333\cancel{3}, \quad L = \underline{74.6666}$$
$$C = 40(37.3333) = 1493.3333$$

   b) [5 marks]Jo's boss now tells her that she must work 50 hours per week. No other amount is acceptable. Show that Jo is worse off because of this rule.

$$U_0 = R^{1/2}C$$
$$U_0 = (37.3333)^{1/2} \times 1493.3333$$
$$= 6.110098 \times 1493.3333$$
$$= 9124.432\cancel{9}$$

If $L = 50$, then $R = 112-50$
$$R = 62$$
$$C = 20(50) = 1000$$

$$U_1 = (62)^{1/2} \cdot 1000 = 7874.00\cancel{7}9$$

So the utility of Jo reduces.

c) Return to the situation in part a). If Jo were to receive non-labour income in the amount of $200 per week, would she work more or less? Explain.

NOW $C = 200 + 20(112 - R)$

$MRS_{R,C} = \frac{\frac{1}{2} R^{-1/2} C}{R^{1/2}} = \frac{\frac{1}{2} C}{R} = \frac{20}{1} \Rightarrow \frac{w}{1} \Rightarrow$

$C = 40R$

sub in BC $40R = 200 + 20(112 - R)$

$60R = 200 + 20(112)$

$60R = 200 + 2240 = 2440$

$R = 40.6666$

$R_1 = 40.6666$

$\Rightarrow$ So as Jo becomes wealthier she takes more leisure ~~hours~~ $\Rightarrow$ from 37.333 to 40.6666

3. [5 marks]Amy is a teacher who earns $6,000,000 during her working life and nothing when she retires. The interest rate between her working life and retirement is 100%. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=Min\{C_P ; 3C_F\}$.

a) Derive Amy's optimal consumption bundle and her level of savings.

$I_P = C_P + S$, $C_F = I_F + S(1+r)$

$S = \frac{C_F - I_F}{1+r}$

$I_P = 6$ mill
$I_F = 0$
$r = 1$

$\boxed{I_P = C_P + \frac{C_F - I_F}{1+r}}$ FVBC

$C_P + \frac{C_F - 0}{2} = 6$ mill

$C_P + \frac{C_F}{2} = 6$ mill

& from U fn: $C_P = 3C_F$, sub in

$3C_F + \frac{C_F}{2} = 6$ mill

$7C_F = 12$ mill

$C_F = \frac{12}{7}$ mill

$C_F = 1.714285$ mill

$\Rightarrow C_P = 3C_F$

$= 3(1.714285)$ mill

$= \underline{5.142857 \text{ mill}}$

$S = I_P - C_P$

$= 6 \text{ mill} - 5.142857 \text{ mill}$

$= \underline{0.857142857 \text{ mill}}$

or $S = \frac{C_F - I_F}{1+r}$

$= \frac{1.714285}{2}$

b) Now suppose the interest rate increases to 200%. Show how this affects Amy's optimal bundle and level of savings. Explain (in words) the income and substitution effects of the interest rate increase.

$r = 2$

Again $C_P = 3C_F$

FVBC $\Rightarrow I_P = C_P + \frac{C_F - I_F}{1+r} \Rightarrow I_P = C_P + \frac{C_F - 0}{3}$

$\Rightarrow 6 \text{ mill} = C_P + \frac{C_F}{3} \Rightarrow 3C_F + \frac{C_F}{3} = 6 \text{ mill}$

$9C_F + C_F = 18 \text{ mill}$

$10 C_F = 18 \text{ mill}$

$C_F = 1.8 \text{ mill}$

$C_P = 3(1.8 \text{ mill})$
$= 5.4 \text{ mill}$

$S = I_P - C_P = 6 \text{ mill} - 5.4 = 0.6 \text{ mill}$

because of the increase in interest she consumes more in the present substituting it from future consumption

c) [5 marks] How would your answer to part (a) change if Amy had preferences given by $U(C_P , C_F) = \text{Min}\{2C_P ; 6C_F\}$? Explain your answer using words and reasoning. Do not solve for the optimal bundle.

Because it is just a rescaling of the utility nothing changes.